A General Low-Parameter 3D Ship Hull Extent Model for Object Tracking

Michael Ernesto López
NTNU, Norway
ernesto.lopez@ntnu.no

Kjetil Vasstein
NTNU, Norway

Edmund Brekke
NTNU, Norway

Rudolf Mester
NTNU, Norway

Annette Stahl
NTNU, Norway

***Abstract*—In autonomous vehicle systems, it is paramount to detect other objects in the vicinity and track their movement. Extended Object Tracking (EOT) provides a convenient framework for tracking objects using high-resolution sensor data by defining models for the object's spatial dimensions (a.k.a. extent). In maritime applications, the objects of interest are mainly other maritime vessels, and these vary greatly in shape and size. This diversity proves to be a challenge for defining general extent models that both give accurate representations for most vessels and that do not depend on a large number of parameters. In this paper, a general three-dimensional low-parameter ship hull model designed for EOT is presented. The presented extent model is constructed by intertwining a polynomial representation along the vertical direction with a frequency representation along the horizontal plane. However, to reduce the dimension of the parameter space without compromising its accuracy, the horizontal frequency representation is modified by performing a Principal Component Analysis (PCA). In particular, this extent representation does not require an underlying discretization grid, which makes the model scalable and therefore well-suited for modeling objects that vary greatly in size.**

***Index Terms*—Extended Object Tracking, Statistical Modeling, Principal Component Analysis**

I. Introduction

Object tracking is the problem of estimating and predicting the movement of objects of interest present in the surroundings over time using the available sensor information. The classical approach to object tracking is to assume that the spatial dimensions of each target can be modeled as a single point. In other words, the target's spatial dimensions, also referred to as its extent, are assumed to be negligible. Hence, in classical point object tracking, the measurements associated with a target are directly related to the movement of the extent-less single point via a measurement and a kinematic model.

In contrast to point target tracking, EOT does not neglect the spatial dimensions of the target and utilizes an extent model to relate the measurements associated with the target to its kinematic state. The addition of an extent model adds complexity to the tracking algorithm as not only the kinematic state variables have to be determined using the available measurements, but also the parameter values for the additional extent model. Furthermore, during estimation, the added task of determining to which points on the target's extent the associated measurements belong to, has to be solved. Although estimating the target's extent may be beneficial in many applications, such as providing safety margins for collision avoidance or helping classify the object and its likely intentions, the main motivation behind introducing an extent model is that it allows for a better prediction of the target's motion in the immediate future by utilizing a more accurate overall model.

Another argument for advocating for EOT instead of the classical Point Object Tracking is that the resolution of modern sensors is so high that the extent-less point assumption breaks down, and it is necessary to define an extent model in order to relate the many available measurements to the target's kinematic state. If we take this argument to heart, then one would also advocate for three-dimensional extent models because the high resolution of modern sensors is in most cases present in all three spatial directions. For example, LiDAR sensors provide three-dimensional point clouds of range measurements with high-resolutions along both the horizontal and vertical directions. However, most extent models found in the literature, especially in the case of maritime applications, are two-dimensional in the sense that the object's movement, its extent, and the measurements are only considered along the horizontal plane. This projection onto the horizontal plane introduces challenges in modelling and estimation. For example, the projection of measurements that actually lie on the surface of the extent may now lie in the interior of the projected two-dimensional extent. The introduced modeling errors may be alleviated by defining ad hoc measurement models. The Random Matrix approach [3] and the measurement model in [9] are examples of such solutions.

Three dimensional extent models are mostly found in urban or automotive applications, where they are used to describe objects that do not vary much in size or shape such as pedestrians, bicycles and automobiles. These objects may be modelled using fixed shape extents such as ellipses, lines or rectangles [4], or by a more general extent model that adapts to the particular features of the object such as the extent models developed in [6] and [10].

One of the few examples of three dimensional extent models for maritime vessels can be found in [2]. Here, a general three-dimensional extent model based on the combination of a Fourier representation along the horizontal plane and

This work was supported in part by the Research Council of Norway through Project 295033 Autosit; in part by DNV; in part by Kongsberg; and in part by Maritime Robotics.

a polynomial representation along the vertical direction is introduced.

The challenges of three-dimensional ship extent modelling arise from the general requirements on extent models and the fact that maritime vessels vary greatly in shape and size. In general, an extent model should depend on as few parameters as possible in order to not overcomplicate the estimation process while also being able to represent the objects of interest as accurately as possible. This compromise leads to several challenges in model design for the particular case of maritime vessels. First, there is the question whether to use an extent model that assumes a fixed shape or not, as fixed size extent models require very few parameters. Experience has shown that fixed shape extent models are ill-suited when applied to maritime vessels (see e.g. [8]). In such an application, the estimation algorithm will try to fit the curvature of the fixed shape model to the part of the vessel that is illuminated by the sensor at a particular time. However, the curvature of the hull may vary greatly from one vessel to another. Therefore, if a fixed shape extent model is used, this may result in a distorted extent estimate that is only accurate for a small part of the extent. Fig. 1 illustrates this phenomenon in the case where an elliptical shape is fitted to point measurements of a ship hull.

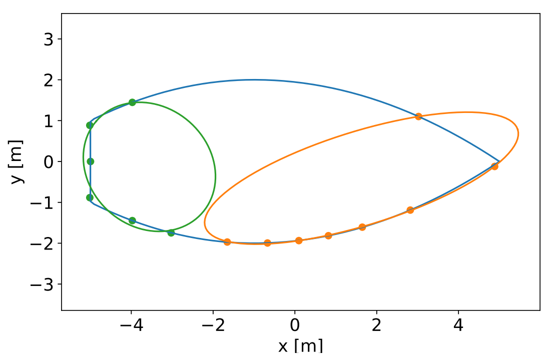


Fig. 1: Results from fitting a fixed ellipse shape (orange and green curves) to point measurements (dots) of a ship extent (blue curve). The elliptical approximations vary greatly depending on which part of the vessel is being illuminated by the sensor.

Another challenge that arises when modelling the extent of maritime vessels is that there is no telling in what type of superstructures one may encounter over their hull when the vessel is of a certain size. Therefore, a general extent model for maritime vessels that also describes their superstructures is not likely to depend on a low number of parameters. However, a practical alternative to modelling the whole vessel is to restrict the modelling task to the hull of the vessel. The reasons behind this approach are that hulls are always present in maritime vessels, they constitute a considerable part of the whole object, and they posses dominant features that may be represented with few parameters. In addition, one can determine the position, orientation and intentions of the whole vessel just by looking at the hull's pose.

Furthermore, we observe that the extent model for maritime vessels should preferably not be defined so that there is a direct relation between the number of models parameters and the number of points in a discretization of the extent. If such direct relation were to exist, then the resolution in the extent approximation will decrease with the maritime vessels size unless the number of parameters is increased. This effect is not desirable, as we would prefer an extent model that is able to represent maritime vessels regardless of their scale with the same amount of parameters. Due to the observations above, we advocate in this paper for an extent model tailored for ship hulls and that scales well with the extent's overall size.

A. Contributions

A general three-dimensional ship hull extent model that provides a sufficient approximation with a low-number of parameters, and that does not depend on an underlying discretization grid is presented. Although elements of the developed model can be found in the literature, such as the use of truncated Fourier series in the horizontal direction and polynomials along the vertical axis (see for example [2]), the use of Principal Component Analysis ([5]) on a representative ship hull data set, and the conscient choice to limit the extent model to the hull of a maritime vessel, makes the resulting extent model a novelty.

Furthermore, it is worth to clarify that the contributions of this paper do not constitute a complete EOT method in itself, but rather ancillary work towards the construction of EOT methods for maritime vessels. The presented ship hull extent model aims to be as general as possible, and may be in principle combined with any kinematic and measurement model. However, the extent model is better suited for high-resolution measurements, such as the ones provided by LiDAR sensors.

II. The 3D Ship Hull Extent Model

For the purposes of this paper, the concept of ship hull is defined so that every point on the hull walls $\mathbf{p}^b$ may be well-represented by a cylindrical coordinate representation of the form

$$\mathbf{p}^b(\theta, h) = \begin{bmatrix} F(\theta, h)\cos(\theta) \\ F(\theta, h)\sin(\theta) \\ h \end{bmatrix} , \tag{1}$$

where $F(\theta, h)$ is the extent's radius function.

In order for the cylindrical representation in (1) to be well-defined, the hull of a maritime vessel is defined as the part of the vessel that spans from the waterline up to a height H with the condition that there exist an upwards pointing vertical axis $\hat{\mathbf{z}}$, so that for each height level $h \in [0, H]$, the corresponding horizontal slice of the maritime vessel's extent is star-convex with respect to the intersection point between the axis $\hat{\mathbf{z}}$ and the horizontal plane at height h.

In addition, it is assumed that the hull is symmetric along a vertical plane that contains the axis $\hat{\mathbf{z}}$. In other words, this assumption models the port and starboard sides of the hull as mirror images of each other, as is the case with nearly all maritime vessels. Under these assumptions, the body frame attached to the hull is defined with axes $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$, where $\hat{\mathbf{x}}$ is in the symmetry plane and it points to the bow of the vessel, and the center of the body frame c is the intersection of the water plane and the axis $\hat{\mathbf{z}}$.

It follows from this definition that the cylindrical representation of the hull walls (port and starboard sides) with respect to this body frame is indeed given by (1), and that the radius function $F(\theta, h)$ is well-defined. In particular, the task of determining the hull is equivalent to determining the values of the radius function $F(\theta, h)$ for $(\theta, h) \in [-\pi, \pi] \times [0, H]$.

Note that $F(\theta, 0)$ corresponds to the values of the hull along the waterline, while $F(\theta, H)$ gives the values of the hull top. Furthermore, the symmetry assumption on the extent implies that the radius function $F(\theta, H)$ is even on the variable θ.

For some maritime vessels, the hull extent definition introduced above might cover almost the whole vessel, while for many other vessels the hull extent might only correspond to a small part of the actual hull. In any case, the introduced hull definition is sufficient for the objectives of target tracking because the hull extent will always correspond to a sizeable part of the vessel, which will be complete along the horizontal direction. Therefore, tracking the hull extent is equivalent to tracking the original maritime vessel.

A. Horizontal Fourier based Model

The first step towards the full three-dimensional ship hull extent model is to model any horizontal slice of the hull, which will be referred to as a two-dimensional hull.

Let us fix a height level $h \in [0, H]$, then the radius function for the corresponding two-dimensional hull is denoted by $f(\theta) = F(\theta, h)$.

Since the extent model should be general and it should not depend on a particular discretization of the extent boundary, the even radius function $f(\theta)$ is approximated by its truncated Fourier series of order N_f, which is given by

$$f(\theta) \simeq \frac{A_0}{2} + \sum_{n=1}^{N_f-1} A_n \cos(n\theta)\,, \tag{2}$$

or rewritten in vector form as

$$f(\theta) \simeq \mathbf{g}(\theta)^T \mathbf{A}\,, \tag{3}$$

with

$$\mathbf{g}(\theta) = \begin{bmatrix} \frac{1}{2} & \cos(\theta) & \cos(2\theta) & \cdots & \cos((N_f-1)\theta) \end{bmatrix}^T \tag{4}$$

$$\mathbf{A} = \begin{bmatrix} A_0 & A_1 & A_2 & \cdots & A_{N_f-1} \end{bmatrix}^T . \tag{5}$$

Equation (3) states that the approximation of the radius function $f(\theta)$ is completely determined by the value of the Fourier coefficient vector $\mathbf{A}$.

Fig. 2 shows examples of simulated two-dimensional hull examples (Fig. 2a), their radius functions (Fig. 2b), and their associated Fourier coefficients (Fig. 2c). Since the radius function is a positive function, the Fourier coefficients will form an alternating sequence of Fourier coefficients A_n, and any truncated Fourier series will lead to an extent approximation that oscillates around the true hull shape as illustrated in Fig. 2d. The oscillations may be mitigated by considering a sufficiently large number of Fourier coefficients. However, choosing such a strategy is unfortunate because two-dimensional hull shapes are not so diverse as to justify such a high dimension in the extent model parameter space.

PCA ([5]) is therefore used on a simulated set of two-dimensional hull shapes in order to ensure a low parameter space dimension, while achieving a sufficiently accurate extent approximation. The dataset is composed of hulls of different shape types, which are illustrated in Fig. 2a. Many examples of these ship hulls are generated by sampling different values of the variables L, B, S, and D, which correspond to the length of the hull, its width, the width of the stern and the point along the longitudinal axis where the hull is widest. The meaning of these variables are illustrated in Fig. 3, and table I contains the range of the chosen variable values. The length of the hull L has been selected as a normalizing unit for the data set of two-dimensional hulls, and it is treated as a measure of the scale of the hull extent. For the PCA the length is assumed to be unity, and the results from the PCA will be later corrected by scaling them with the length L. Ideally, L should then only take the value of 1.0 for every hull in the data set, but the range from 0.8 to 1.2 was chosen in order to compensate for possible estimations errors in the length of the vessel when performing tracking.

TABLE I: Two-dimensional hull data set

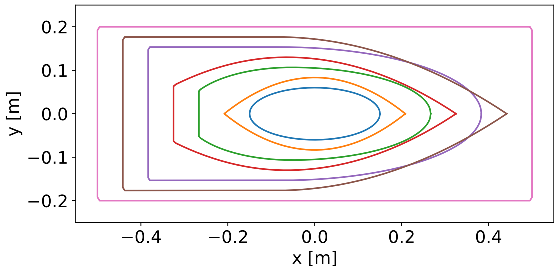
	Variable ranges			
	L	W/L	D/L	S/W
Min	0.8	0.2	0.2	0.2
Max	1.2	0.8	0.8	0.8

The total number of generated two-dimensional ship hull models is 4375. For each two-dimensional hull, the Fourier coefficient vector $\mathbf{A}$ for the truncated series with $N_f = 64$ components is calculated. The number of frequency components $N_f = 64$ was chosen because this number of components is sufficiently large to provide an accurate approximation of the hulls, and the number is a potency of 2, which enables fast Fourier transform computations.

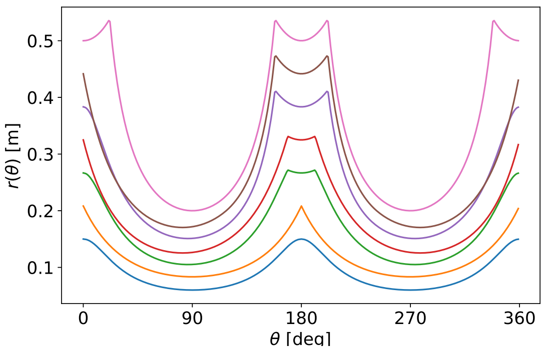
The PCA is applied to the set of generated Fourier coefficient vectors $\mathbf{A}$. Fig. 4 shows the absolute value of the correlation coefficients between these vectors. Since there are non-negligible off diagonal correlations between the Fourier coefficients, the PCA may provide a sufficient approximation to the set of Fourier component vectors with a reduced number of parameters.

For a fixed number of principal components N_{PC} with $N_{\text{PC}} \leq N_f = 64$, the PCA approximates the data set of Fourier coefficient vectors $\mathbf{A}$ by an affine space of dimension N_{PC} of the form

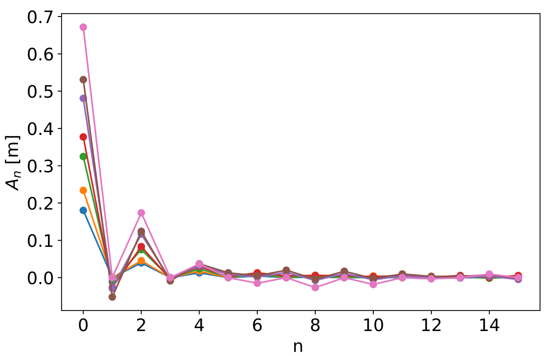
$$\mathbf{A} \simeq \boldsymbol{\mu} + \mathbf{M}\mathbf{e}\,, \tag{6}$$



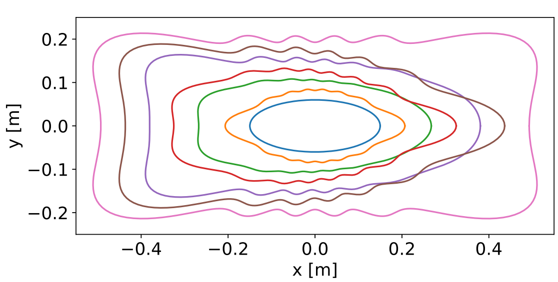
(a) Body coordinates.



(b) Radius functions.



(c) Fourier coefficients.



(d) Body coordinates of the truncated Fourier series approximation with $N_f = 16$.

Fig. 2: Two-dimensional ship hull examples.

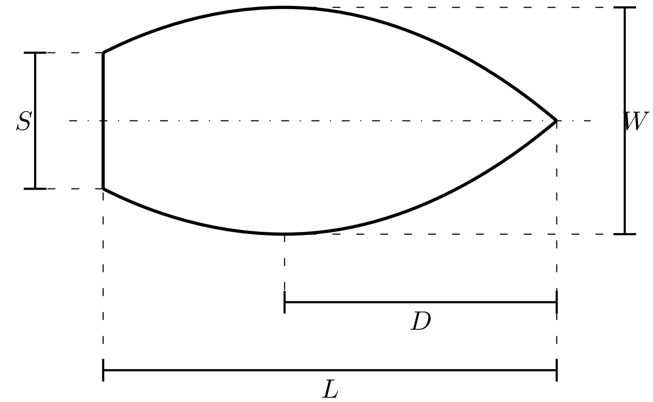


Fig. 3: Geometric meaning of the variables L, W, D and S used for generating the data set of two-dimensional hull extents.

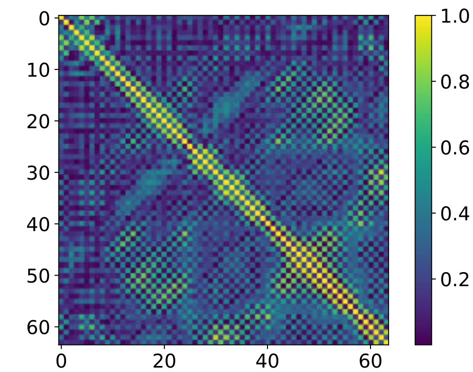


Fig. 4: Absolute value of correlation coefficients between the Fourier component vectors $\mathbf{A}$ of the generated two-dimensional hulls.

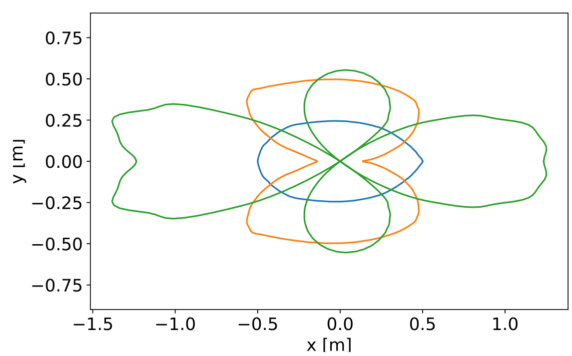
where $\boldsymbol{\mu}$ is the mean of the considered Fourier coefficient vectors, $\mathbf{M}$ is a N_f by N_{PC} matrix, whose columns are the N_{PC} eigenvectors associated to the largest eigenvalues of the data set covariance matrix, and the vector $\mathbf{e}$ is the parameter vector of length N_{PC}.

By inserting (6) into (3), and by taking into account that the PCA was performed assuming that the two-dimensional hulls had unit length, the final two-dimensional hull extent model is given by

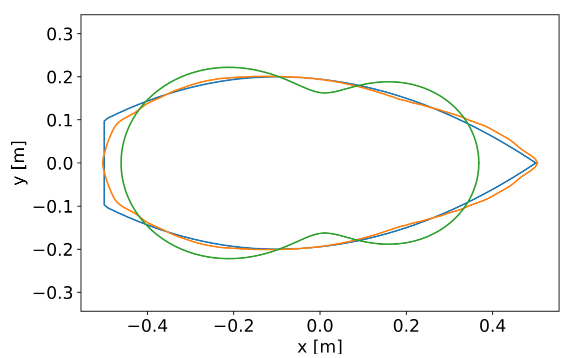
$$f(\theta) \simeq L\mathbf{g}(\theta)^T (\boldsymbol{\mu} + \mathbf{M}\mathbf{e}) . \tag{7}$$

The mean vector $\boldsymbol{\mu}$ and the component matrix $\mathbf{M}$ have been learned from the generated data set of two-dimensional hulls, and they are part of the extent model. On the other hand, the length L and the parameter vector $\mathbf{e}$ constitute the extent parameters, and their values characterize the particular two-dimensional hull.

Fig. 5 illustrates some details of the Fourier and PCA based extent model for two-dimensional hulls. Fig. 5a shows the extents associated to the mean $\boldsymbol{\mu}$ and to the two first principal components in $\mathbf{M}$. Note that the first principal component mostly captures the details of the sides of the extent, while the second principal component captures the details of the bow and the stern. Fig. 5b compares the extent approximations for the PCA based model (7) and the direct truncated Fourier series (3) for a particular hull example using the same number of parameters. The PCA based model provides a better approximation to the true extent and with less oscillations than the truncated Fourier series, and this is the general case and not only for this particular example.



(a) Body coordinates associated to mean $\boldsymbol{\mu}$ of data set (blue) and body coordinates associated to the first and second principal components (orange and green, respectively).



(b) Two-dimensional extent example (blue), its corresponding truncated Fourier series approximation with $N_f = 4$ (green) and its PCA approximation with $N_{\text{PC}} = 4$ (orange).

Fig. 5: The two-dimensional Fourier-PCA extent model.

B. Three-dimensional Model

The final three-dimensional extent model for ship hulls is constructed by intertwining the two-dimensional ship hull model developed in the previous subsection (7) along the vertical direction using a finite number of basis functions on the interval $[0, H]$. Let $h_0(h), \cdots, h_{N_h-1}(h)$ denote those basis functions.

In contrast to the diversity that ship hulls may present along any horizontal plane, these extents do not vary as much along the vertical direction. In most cases, any vertical section of the hull side walls above the waterline will give curves that seem parabolic and may be parameterized with a low number of parameters. Therefore, no PCA or similar parameter dimension reduction technique is applied along the vertical direction.

Hence, the deviation between the extent radius function $F(\theta, h)$ and the radius function defined by the mean $\boldsymbol{\mu}$, i.e $\mathbf{g}(\theta)^T\boldsymbol{\mu}$, is approximated by a linear combination of all possible products between the principal components from the PCA, i.e. the columns of $\mathbf{M}$, and the basis function h_i, which gives

$$F(\theta, h)/L - \mathbf{g}(\theta)^T\boldsymbol{\mu} \simeq \mathbf{g}(\theta)^T\mathbf{M}\mathbf{E}\mathbf{h}(h) \tag{8}$$
$$\Rightarrow \qquad F(\theta, h) \simeq L\mathbf{g}(\theta)^T\left(\boldsymbol{\mu} + \mathbf{M}\mathbf{E}\mathbf{h}(h)\right), \tag{9}$$

where

$$\mathbf{h}(h) = \left[h_0(h) \cdots h_{N_h-1}(h)\right]^T. \tag{10}$$

and $\mathbf{E}$ is a N_{PC} by N_h matrix.

Equation (9) provides the three-dimensional ship hull extent model. Once again, the mean vector $\boldsymbol{\mu}$ and the principal component matrix $\mathbf{M}$ are part of the model, and their values have been learned from analyzing two-dimensional hulls. The parameters of the three-dimensional model are the length of the ship L and the matrix $\mathbf{E}$. In particular, the extent model depends on $N_{\text{PC}} \cdot N_h + 1$ parameters.

There are many options for the basis functions h_i. However, we once more insist on using representations that do not depend on a particular discretization of the interval $[0, H]$. Furthermore, due to the parabolic nature of vertical sections of hull walls, polynomial basis functions are chosen. More precisely, Chebyshev polynomials of the first kind combined with a linear transformation from the interval $[0, H]$ onto the interval $[-1, 1]$ are selected. Hence, the basis functions used in this paper are given by

$$h_i(h) = c_i\left(\frac{2h}{H} - 1\right), \tag{11}$$

where

$$c_0(u) = 1 \tag{12}$$
$$c_1(u) = u \tag{13}$$
$$c_i(u) = 2uc_{i-1}(u) - c_{i-2} \quad \text{for } i \geq 2, \tag{14}$$

$h \in [0, H]$ and $u \in [-1, 1]$.

In order to study the accuracy of the extent model as a function of the number of principal components N_{PC} and the number of basis functions N_h, a data set of three-dimensional computer-aided design (CAD) models of maritime vessels was collected. This set consists of 198 maritime vessels, whose CAD models were open access and available in the software Delftship. The collected data set covers many different ship types and sizes. However, it should be noted that the vessel examples are biased towards large ships, such as container ships. A considerable amount of manual work was necessary to extract the hull from these models and to simulate realistic waterlines, as every model not only represents a different vessel, but it was also designed by a different author with different design choices. For every CAD model, three different waterline levels were simulated, and the three-dimensional model in (9) was applied to approximate the vessel's hull.

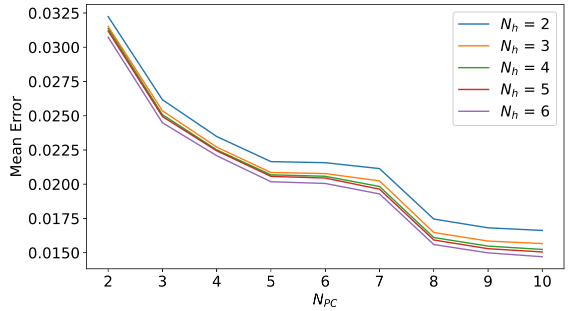
For each combination of CAD model and waterline level, the approximation error E is the mean of the deviation between the actual radius function F_{true} and its approximation F_{est} over a discretization of $[-\pi, \pi] \times [0, H]$, i.e.

$$E = \frac{1}{N_d} \sum_{n=1}^{N_d} \left| F_{\text{true}}(\theta_n, h_n) - F_{\text{est}}(\theta_n, h_n) \right| \,, \qquad (15)$$

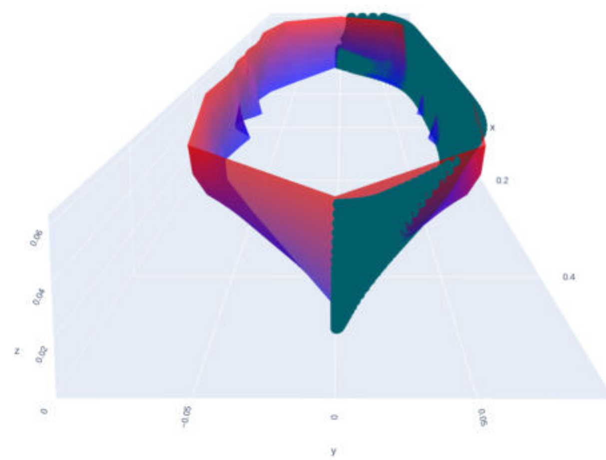
where $\{(\theta_n, h_n)\}_{n=1}^{N_d}$ is a fine discretization of $[-\pi, \pi] \times [0, H]$. Moreover, the overall mean error is the mean of the approximation error E over all possible combinations of CAD models and waterline levels.

Fig. 6a shows the mean error of the radius function approximation for different values of N_{PC} and N_h. We observe that considering up to 5 principal component gives the best return in terms of accuracy. Of course, the higher the number of parameters is, the better the approximation becomes. However, in our experience, an extent model with $N_{\text{PC}} = 5$ and $N_h = 3$ already gives very good approximations to the actual extent. Fig. 6b shows an example of such approximation in body coordinates.

The three-dimensional extent model with $N_{\text{PC}} = 5$ and $N_h = 3$ has 16 parameters. This number is comparable to the number of parameters needed in two-dimensional general extent models found in the literature (see e.g. [1] or [7] or [9]), and it is considerably less than the number of parameters used in other general three-dimensional extent models (see e.g. [2]).



(a) Mean error of radius function approximation for different values of N_{PC} and N_h.



(b) Three-dimensional ship hull example (only port side is shown in green) and its approximation using an extent model with $N_{\text{PC}} = 5$ and $N_h = 3$ (blue and red surface).

Fig. 6: Three-dimensional ship hull extent model.

III. Conclusions

Modeling the extent of maritime vessels with a low number of parameters is a challenging task due to their great variation in size and shape and due to the presence of superstructures. In this paper, it has been shown that if the modeling task is restricted to the hull of the maritime vessel, then it is possible to derive a three-dimensional ship hull model that depends on a low number of parameters and that provides sufficiently accurate approximations to the true extent. The number of parameters used is comparable to the number of parameters used in general two-dimensional extent models found in the literature.

The three-dimensional model is constructed by intertwining a frequency representation modified using PCA along the horizontal plane with a basis of polynomial functions along the vertical direction. These polynomials are based on Chebyshev Polynomials of the first kind. Although, the initial results for this extent model are promising, it remains to be seen whether other representations along the horizontal or vertical directions may provide a better three-dimensional extent model.

In order to test the quality of the current three-dimensional extent model and future others, a larger and unbiased set of three-dimensional ship CAD models for validation needs to be developed.

Acknowledgment

We would like to thank Martin L. Baerveldt, Nicholas Dalhaug and Trym A. Nygård for their input on different aspects of extent modelling for maritime applications.

References

[1] L. M. Baerveldt, M. E. Lopez, and E. F. Brekke. "Extended Target PMBM Tracker with a Gaussian Process Target Model on LiDAR Data". In: *2023 26th International Conference on Information Fusion (FUSION)*. IEEE. 2023.

[2] T. Baur et al. "Shape Tracking using Fourier-Chebyshev Double Series for 3D Distance Measurements". In: *2023 26th International Conference on Information Fusion (FUSION)*. IEEE. 2023.

[3] K. Granström and M. Baum. "A Tutorial on Multiple Extended Object Tracking". In: *Authorea Preprints* (2023).

[4] K. Granström, M. Baum, and S. Reuter. "Extended Object Tracking: Introduction, Overview, and Applications." In: *Journal of Advances in Information Fusion* 12.2 (2017).

[5] I. Jolliffe. *Principal Component Analysis*. Springer Series in Statistics. Springer, 2002. ISBN: 9780387954424.

[6] M. Kumru and E. Özkan. "Three-dimensional extended object tracking and shape learning using Gaussian processes". In: *IEEE Transactions on Aerospace and Electronic Systems* 57.5 (2021), pp. 2795–2814.

[7] M. E. Lopez et al. "Multiscan Shape Estimation for Extended Object Tracking". In: *2023 26th International Conference on Information Fusion (FUSION)*. IEEE. 2023.

[8] K. A. Ruud, E. F. Brekke, and J. Eidsvik. "LIDAR Extended Object Tracking of a Maritime Vessel using an Ellipsoidal Contour Model". In: *2018 Sensor Data Fusion: Trends, Solutions, Applications (SDF)*. 2018.

[9] N. Wahlström and E. Özkan. "Extended Target Tracking using Gaussian Processes". In: *IEEE Transactions on Signal Processing* 63.16 (2015).

[10] R. Wang et al. "Directshape: Direct Photometric Alignment of Shape Priors for Visual Vehicle Pose and Shape Estimation". In: *2020 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE. 2020.